Equations (2) and (3a) are sufficient to describe families of curves for various values of Z_0 and y as a function of r and x as circles and lemniscates, respectively. A solution for the parameters y and Z_0 is restricted to all values of r and x that lie inside the $Z_0=0$ and $Z_0=\infty$ circles in order to make the desired value of Z_0 positive and real. It is interesting to note that the parameter y is infinite for purely resistive loads leading to the usual quarter-wave transformer design. For complex load impedances, however, the overall length of the transformer is generally less than quarter wavelength when the range of y is between zero and infinity.

The proposed design procedure may therefore be summarized by the following steps.

1) Normalize R_L and X_L with respect to R to obtain r and x.
2) Locate the normalized impedance point and find the particular circle and lemniscate that intersect at the same point noting the labeled values of Z_0 and y.
3) Compute Z_0' and d. If the resulting value of d is negative, add a half wavelength section of characteristic impedance Z_0'.
4) If the impedance point in step 2) lies outside the $Z_0=0$ or $Z_0=\infty$ circles, a section of characteristic impedance R must be inserted between the load and the transformer. The length of this section is obtained by moving a minimum distance towards the generator, along a constant VSWR circle, to a new load point located inside the $Z_0=0$ or $Z_0=\infty$ circles. The remaining procedure is identical to steps 2) and 3).

The advantage of the proposed design over the quarter-wave transformer is the possible reduction in the insertion loss which is primarily due to the smaller number of junctions and shorter length of the matching section. To illustrate this advantage in a practical case, a complex load impedance of $40.7-j220$ ohms, corresponding to a 45.72-meter dipole with length-to-diameter ratio of 1800 and operating at 2.6 MHz, was selected. The feed-line characteristic impedance was 30 ohms and the values of d and Z_0' were found to be 9.923 meters and 370 ohms, respectively. The quarter-wave design required a 30-ohm, 26.434-meter-long line in series with the load and a 28.846-meter-long quarter-wave section with a characteristic impedance of 4.6 ohms. The reduction in the insertion loss is apparent from the shorter length required. However, the narrow bandwidth for both transformers, as shown for this case in Fig. 2, may not be a serious disadvantage where, for instance, the dipole corresponds to a transmitting antenna.

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Nonsymmetrical Coupled Lines of Reentrant Cross Section

Frequently, UHF and microwave TEM components require tightly coupled transmission lines, directional couplers [1] of greater than −8 dB coupling, narrow-to-moderate-bandwidth bandpass filters [2], moderate-to-wide-bandwidth stopband filters [3], and 90-degree phase shifters [4], [5] are some examples. The novel geometry of the directional coupler with reentrant cross section [6]–[9] can result in very tightly coupled transmission lines and has been proved practical and useful. Recently, the basic geometry and the corresponding design equations for directional couplers with reentrant cross section were extended [10] to provide a slightly more general geometry capable of even greater coupling with practical physical dimensions. However, in the aforementioned cases, only symmetrical geometries and their related equations were presented. In this correspondence, the basic reentrant cross-sectional geometry and associated equations are generalized to include the nonsymmetrical case which is important for nonsymmetrical directional couplers [11] and coupled strip transmission-line filters requiring coupled nonsymmetrical lines [2].

Nonsymmetrical coupled lines of reentrant cross section are shown in coaxial, stripline, and microstrip form in Fig. 1(a), (b), and (c), respectively. The geometries of Fig. 1(a) and (b) were previously presented in symmetrical form [6]–[9]; the microstrip geometry of Fig. 1(c) is new and is presented here in the general nonsymmetrical form.

Application of the coupled-lines configuration to a specific problem requires specification of the even- and odd-mode admittances (or capacitances) of the coupled lines [1]. For line a, these admittances are denoted by Y_{oe}^a and Y_{oo}^a, respectively; for line b, they are denoted by Y_{oe}^b and Y_{oo}^b, respectively. These parameters are generally determined from a set of design equations for the particular problem at hand. The admittances are related to the normalized capacitances by

$$C/\varepsilon = \frac{376.7Y}{\sqrt{\varepsilon_r}}, \tag{1}$$

where

1) C/ε = normalized capacitance, which is dimensionless and independent of the permittivity of the medium;
2) Y = the unnormalized admittance in mhos;
3) ε_r = the relative dielectric constant of the medium;
4) ε = the permittivity of the medium in farads/unit length.

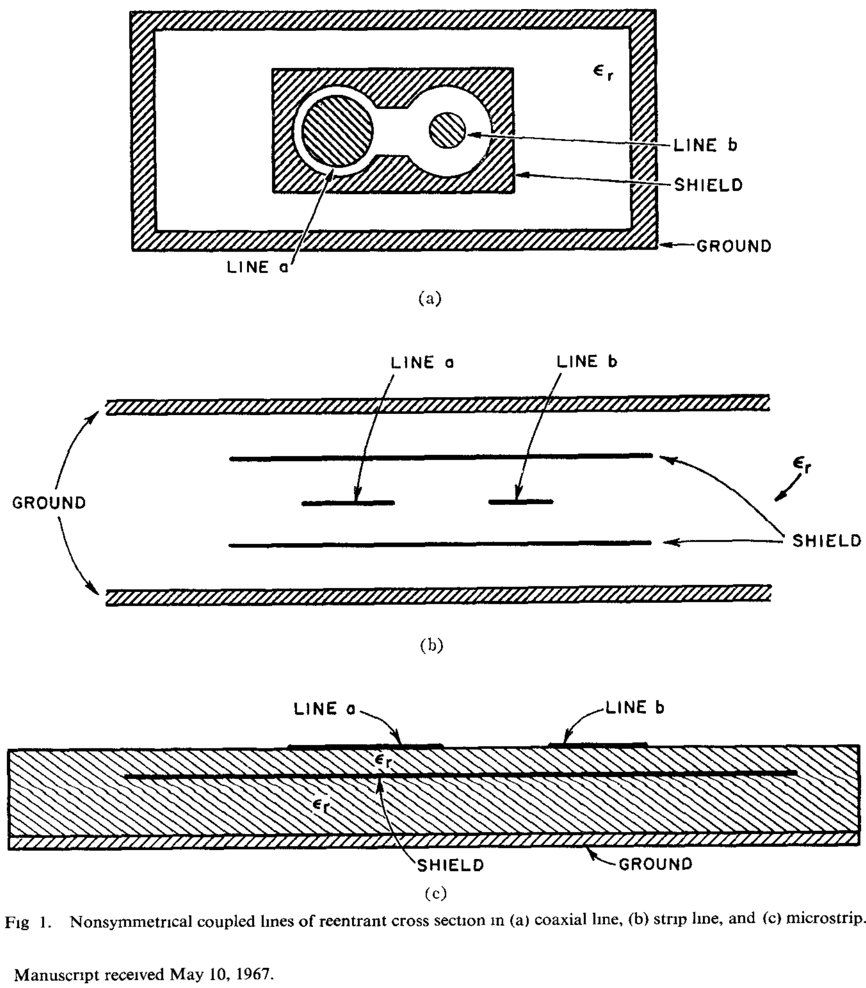

Fig 1. Nonsymmetrical coupled lines of reentrant cross section in (a) coaxial line, (b) strip line, and (c) microstrip.

Manuscript received May 10, 1967.

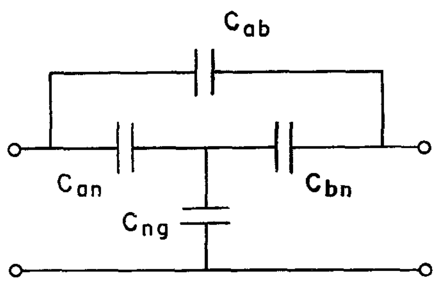

Fig. 2. Capacitance network at an arbitrary cross section of coupled lines.

The static capacitance network of the coupled lines at an arbitrary cross section of the geometries of Fig. 1(a), (b), and (c), is given in Fig. 2. In this figure, C_{an} is the capacitance between line a and the shield; C_{bn} is similarly defined for line b; C_{ab} is the mutual capacitance between lines a and b; and C_{ng} is the total capacitance between the shield and ground. It is assumed that the shield is long enough that direct capacitance between line a or b and ground is negligible.

The relationship between the several static capacitances and the even-mode, odd-mode admittances (or capacitances) are given in the following equations. Define the parameters α, β, γ as follows:

$$\alpha = \left\{ \frac{376.7Y}{\sqrt{\varepsilon_r}} \; Y_{oe}^a \right\}, \tag{2}$$

$$\beta = \left\{ \frac{376.7}{\sqrt{\varepsilon_r}} \; Y_{oe}^b \right\}, \tag{3}$$

$$\gamma = \left\{ \frac{376.7}{\sqrt{\varepsilon_r}} \left[\frac{Y_{oo}^b - Y_{oe}^b}{2} \right] - \frac{C_{ab}}{\varepsilon} \right\}. \tag{4}$$

Choose a convenient value[1] for C_{ab}/ε. Then,

$$C_{an}/\varepsilon = \alpha + \gamma + (\alpha/\beta)\gamma, \tag{5}$$

$$C_{bn}/\varepsilon = \beta + \gamma + (\beta/\alpha)\gamma, \tag{6}$$

$$C_{ng}/\varepsilon = \alpha + \beta + (\alpha/\beta)\gamma. \tag{7}$$

To relate the normalized capacitances C_{an}/ε, C_{bn}/ε, and C_{ab}/ε to geometric dimensions using graphs or experimental methods, use the network of the shield and coupled lines a and b, considered alone (i.e., without the ground plane), as a coupled pair of transmission lines. To relate C_{ng}/ε to geometric dimensions, use the network of the ground planes and shield, considered as a single transmission line (i.e., the shield is excited in the even mode with respect to the ground plane).

As a check on (5), (6), and (7), it can be verified that for symmetrical lines (i.e., $\alpha=\beta$) the equations reduce to those previously given [10]. Furthermore, for symmetrical lines and also $C_{ab}/\varepsilon=0$, the equations are equivalent to those given by Cohn [6] for the case of directional couplers.

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[1] C_{ab}/ε must be small enough for C_{an}/ε, C_{bn}/ε, and C_{ng}/ε in (5), (6), and (7) to be positive. Typically, $0 \leq C_{ab}/\varepsilon \leq 3$.

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Circulator Action at 140 GHz in a Semiconductor Loaded Waveguide Junction

***Abstract*—Circulator action has been observed in an *E*-plane "Y" waveguide junction having an InSb rod aligned with the principal symmetry axis of the junction. With the junction cooled to 77°K, typical isolations of 14–17 dB have been obtained at 138.5–140.1 GHz by applying a 10.5-kG magnetic field along the symmetry axis. Standing wave ratio was 1.92 at the input port but showed an unexplained decrease to 1.27 at a temperature slightly above 77°K. When the apparatus was warmed up to room temperature, only reciprocal behavior was observed.**

Manuscript received May 20, 1967. This work was supported by the Advanced Research Projects Agency, Department of Defense, through the Materials Research Center, Northwestern University, Evanston, Ill.

In this correspondence we report our observation of circulator action in a semiconductor loaded *E*-plane "Y" junction. The experiment was carried out at a wavelength of 2.2 mm, and typical isolations of 14–17 dB were observed. The semiconducting sample was an indium antimonide post aligned with the principal symmetry axis of the junction.

The experiment was suggested by previous work on a gas plasma[1] in which a low-density plasma was similarly located in a junction and nonreciprocity was observed at 3-cm wavelengths.

The basic thesis is that the anisotropic material removes the degeneracy associated with the two counter-rotating polarizations along the principal symmetry axis of the junction. A measure of the anisotropy is the value of the off-diagonal component of the conductivity tensor[2]

$$\sigma_{31} = \frac{\mu B \sigma_0}{(\mu B)^2 + (1 + j\omega\tau)^2},$$

where μB is the product of the mobility and static magnetic flux density, σ_0 is the dc conductivity, and $\omega\tau$ is the product of observing frequency and relaxation time.

A survey of high-mobility semiconductor materials indicated that indium antimonide would be a likely candidate for this experiment. A computation of the complex off-diagonal component of the tensor with the following values:

$$\mu = 2.6 \times 10^5 \text{ cm}^2/\text{V}\cdot\text{s},$$
$$\tau = 2.07 \times 10^{-12} \text{ s},$$
$$\sigma_0 = 95.5\ (\Omega\cdot\text{cm})^{-1}$$

has been carried out, and a graph of the complex component is shown in Fig. 1. The graph contains both the real and imaginary parts of the conductivity tensor element and the range of μB over which the experiment was carried out.

A rod of indium antimonide was placed in the center of a milled brass "Y" junction and placed in a styrofoam Dewar. At 77°K, experiments were performed to measure the insertion loss between an input port and two output ports as a function of the magnetic field.

The results of this experiment are shown in Figs. 2 and 3. The zero field losses in the empty junction and the connecting guides have been subtracted from the overall insertion loss. The insertion loss in the figure therefore represents only the contribution of the anisotropic semiconductor. The discrepancy in zero field insertion loss between the two graphs is probably due to misalignment of the rod.

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